

chain-order_transitivity^{13,45}

$\forall es:ES, Cmd:Type, In:AbsInterface(Cmd), isupdate:(Cmd \rightarrow \mathbb{B}), Sys, Out:AbsInterface(Top).$
 $(E(In) \subseteq_r E(Sys))$
 $\Rightarrow (\forall f:sys\text{-antecedent}(es;Sys), chain:(E(Sys) \rightarrow (Id\ List)).$
 $\quad chain\text{-consistent}(f;chain) \Rightarrow (\forall x, y, z:Id. x \ll y \Rightarrow y \ll z \Rightarrow x \ll z))$